Probabilistic analysis of excavation-induced damages to existing structures

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A simplified probabilistic methodology is proposed to evaluate excavation-induced damage to adjacent existing buildings from the results of a numerical model of the boundary value problem. To this aim, the probabilities of different damage states are computed as a function of both relevant input parameters, considered as independent random variables, and the relative position between the excavation and the buildings. The proposed approach can be defined as a reliability-based design procedure based on monovariate or multivariate probabilistic analyses, respectively for reinforced concrete and masonry buildings. The procedure is applied to a case-study: the design of an underground station, part of a new subway line in the city of Naples (Italy), for which a large open-pit excavation has been designed.

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1. Introduction

Empty spaces in urban areas are extremely rare nowadays and the growing number of urban activities is leading, more and more often, to the use of underground space for installing urban services, transportation infrastructure, parking areas or other engineering works which require the design of deep excavations in densely populated districts or in areas characterized by the presence of historical buildings and cultural heritage sites. Excavations inevitably induce significant changes both in the stress and strain fields of the soil around them and, therefore, displacements to adjacent structures and infrastructure [1–3]. Many different approaches to estimate excavation-induced vertical and horizontal displacements have been proposed in the literature. They can be classified in two classes: empirical methods, e.g. [4–6], and numerical methods, e.g. [7–9]. As for the displacement-induced damages to existing structures located around the excavation area, their estimation is often carried out by using damage indices and functions, e.g. [10–14]. With reference to these issues, the use of reliability-based design methodologies has been greatly developed in the recent past. Indeed, such approaches are nowadays the basis of the most important geo-structural codes worldwide, e.g. [13,15–17], and a number of new methodologies have been proposed to estimate the structural damage by using theoretical or empirical data on settlements and by introducing new potential damage criteria within a probabilistic framework [18–21].

The present paper provides a contribution towards defining simplified probabilistic methods that tackle the complex problem of risk assessment related to the construction of underground structures in urban areas. To this aim, a probabilistic procedure is herein proposed to evaluate the effects of deep excavations to adjacent buildings on the basis of appropriately defined damage functions, which are related to the typology of the affected structures and consider the structural response to the foundations’ settlements and distortions at different distances from the excavation front. Within the procedure, the probabilities of different damage states are computed as a function of relevant input parameters, considered as independent random variables, and of the relative position between the excavation and the building. The proposed procedure, which can be defined as a reliability analysis based on monovariate or multivariate probabilistic analyses, depending on whether the damage criteria refer to a single or to more than one deformation parameters, is applied to a case-study: the design of an underground station, part of a new subway line in the city of Naples (Italy), for which a large open-pit excavation is planned. This application allows a simplified probabilistic evaluation of the effects of the excavation for adjacent buildings located within the computed settlement trough.

2. Ground movements and damage to affected buildings

The estimation of the ground movements induced by deep excavations is a complex geotechnical task, as the response of an
excavation system is always influenced by numerous factors, such as the geometry of the excavation, the geometrical and structural characteristics of the support system, the stiffness and strength characteristics of the soil layers, the construction procedures and phases [22]. Geotechnical analyses aimed at computing the deformations induced in the soil by excavations can be grouped in two classes: (i) empirical (or semi-empirical) methods, (ii) numerical approaches to the boundary value problem. Common to all methods, the ground settlements decrease moving away from the excavation front and become negligible at a distance between 1 and 3 times the excavation depth. Empirical methods are undoubtedly useful for a first estimation of the ground movements induced by deep excavations. Yet, for a detailed analysis of a boundary-value problem, the project-specific characteristics of the excavation system should be taken into account and, therefore, the use of numerical methods that model the soil as a deformable continuum (e.g. Finite Element models) is needed.

Different criteria to evaluate the effect of soil movements on building’s foundations, along with the related damage thresholds, have been proposed in the literature. The numerical thresholds or limit domains are often a function of the building’s use and/or of the type of foundations, consistently with different potential damage levels. According to Ou et al. [23], factors such as the type of foundation, the size of foundation, the length of the side of the excavation and the shape of the settlement profile may affect the building performance during excavation. Thus, information regarding a building’s location relative to the settlement influence zone is helpful in planning building protection measures during excavation. Schuster et al. [20], based on previously developed evaluation criteria, establish a serviceability limit state for reliability analysis of building damage caused by excavation. Bryson and Kotheimer [24] present computed building responses at dates corresponding to observations of cracking and discuss of strain levels in infill panel walls where cracking was observed and in panels where cracking was not observed.

The main deformation parameters used to quantify the movements induced on building’s foundations are reported in Fig. 1. Two main categories of damage criteria may be identified, according to whether they refer to a single or to more than one deformation parameters: damage indices and multi-parameter functions. Several damage thresholds have been proposed in the literature with reference to absolute or differential settlements but substituted the rotation parameter \( \beta \) with the deflection ratio \( \Delta L / L \). Both functions define five homogenous damage zones.

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The first step of the procedure, i.e. the definition of the numerical analysis of the supported deep excavation system (e.g. Finite Element simulation), requires the deployment of the following three models: (i) the geotechnical model, defined taking into account the relevant soil layers, their constitutive behavior and the geotechnical parameters needed to characterize them; (ii) the model of the excavation system, which depends on the geometrical characteristics of the excavation and on the construction procedures (i.e. construction phases and technologies); (iii) the model of the structures affected by the excavation-induced displacements field, which depends on the geometrical characteristics of the excavation as well as on the typology of the affected buildings and on the structural parameters needed to characterize them. Based on these models, the stress and strain field in the area affected by the excavation can be numerically computed at different construction stages to explicitly take into account the soil–structure interaction.

The second step of the procedure consists of a parametric analysis, which considers the model parameters most affecting the system response, aimed at defining the most adequate prediction of the simplified probabilistic estimation of excavation-induced damages to existing structures

A number of new methodologies have been recently proposed to estimate, within a probabilistic framework, the damage to structures adjacent to excavations. Boone [33] and Boone et al. [34] present an approach, intended as a first-order method for damage assessment, relying on ground movement profile geometry, structure geometry and design, strain superposition, and critical strains of building materials. Hsiao et al. [18,19] consider the excavation-induced settlements, which are calculated using a semi-empirical model called KJHH, as the load in the context of reliability analyses and compare them with tolerable settlements, considered as the resistance corresponding to a violation of serviceability requirements. Schuster et al. [20,21] present a simplified model for evaluating the damage potential of a building adjacent to a braced excavation based on a probabilistic assessment of the excavation-induced building damage potential, within which the uncertainty of the model results is computed by applying Bayesian mapping and reliability techniques.

The simplified probabilistic methodology proposed in this paper differs from previous studies as it provides a probabilistic evaluation of the excavation-induced damage to adjacent existing buildings from the results of a numerical model of the boundary value problem given: the typology of the structures, appropriately defined damage functions or thresholds, the relative position between the excavation and the structures. The proposed approach can be defined as a reliability-based design procedure based on monovariate or multivariate probabilistic analyses. A flow chart of the procedure is presented in Fig. 2.

Fig. 1. Main deformation parameters used to quantify the movements induced on a building’s foundations (modified from Eurocode 7 [13]).
model of the boundary value problem. The number of numerical simulations to perform within this phase depends on the characteristics of the previously defined models, more precisely on the uncertainty related to the values of the input parameters of the numerical analysis, as well as on the structural response which is relevant for estimating the damage on the structures, usually the relative settlements and/or rotations of the buildings foundations. At the end of this phase, engineering judgment is necessary to evaluate the results of the analysis in order to select the model that most appropriately predicts the behavior of the excavation system. To this aim, a comparison between the most relevant results of the numerical model (e.g. displacement field of the buildings' foundations) and the corresponding estimations based on empirical or semi-empirical approaches may be helpful.

Within the third step of the procedure, the prediction model selected within the previous phase is used for a probabilistic evaluation of the numerical results. This analysis requires the probabilistic characterization of the model input parameters which are most relevant for the system response as well as the definition of the probabilistic procedure to be used. Regarding the first issue, most of the model uncertainty generally depends on the uncertainty related to the geotechnical model, and in particular on the estimation of the hydraulic conductivity of the soil layers and of the stiffness and strength parameters to be used in the soil constitutive relationships. Regarding the second issue, a Point Estimate Method analysis [35,36], is suggested for estimating the probability density functions of the relevant model results. In the Point Estimate Method (PEM), the geotechnical parameters are modeled as random independent variables characterized by a normal distribution of the probability density function and, therefore, only their mean (i.e. characteristic) values and their variances (or alternatively their coefficients of variation or standard deviations) need to be estimated. To this aim, besides using the results of the geotechnical characterization based on in situ and laboratory tests,
suggestions from the literature can also be profitably used [37–39]. If the random variables are correlated, a correlation matrix also needs to be defined. However, as it is very difficult to establish appropriate coefficients of correlation among geotechnical parameters, it is suggested to only consider independent and uncorrelated input parameters within the analysis. The total number of simulations to be run for a PEM analysis is $2^M$, $M$ being the total number of relevant geotechnical parameters. Being the input...
parameters independent and uncorrelated, the results of all these simulations are to be considered equiprobable and can be easily used to compute the probability density functions of the relevant model results at various distances from the excavation.

Finally, the fourth step of the procedure consists of the evaluation of the probability of damage of the affected buildings at different locations within the excavation-induced settlement trough (i.e. at different distances from the excavation). The results of this probabilistic analysis are functions both of the probability monovariate or multivariate density functions computed in the previous phase and of the shape and values of appropriately defined damage functions taking into account the structural typology of the buildings. Once again, at the end of this phase, engineering judgment needs to be applied to evaluate the results of the analysis in light of appropriately defined damage criteria and to finally decide whether the modeled excavation system is appropriately designed or whether some of the design choices need to be re-evaluated.

4. Analysis of a case study: the excavation system of a subway station in Naples (Italy)

The construction of a new subway line in Naples, which extends along the city’s coastal border, includes the design of different subway stations for which open-pit deep excavations are to be carried out. Among them is the excavation system which is herein analyzed as a case study to apply the procedure proposed in the previous paragraph. The main features of the system are: 23.6 × 85.5 m^3 rectangular-shaped excavation pit; 28 m maximum excavation depth, H; multi-propped 50 m deep T-shaped reinforced concrete slurry walls; and the excavation edge is 16.5 m away from a building’s group having historic value and protected as cultural heritage (Figs. 3 and 4).

4.1. Finite element model of the boundary value problem and parametric analysis

The excavation is planned to be carried out employing a top-down method with reinforced concrete slurry walls permanently

| Table 1 |
| Characteristics of excavation support system. |
| Support system | Dimensions | EI (kN m^2/m) | EA (kN/m) |
| T-shaped r. c. slurry walls | 60 m long | 5.46 m² | 5.8E+06 | 3.4E+07 |
| r. c. slab | Thickness = 1.8 m | |
| First level of struts | 12 struts, Φ800, thickness = 20 mm, pitch = 5 m | 1.8E+06 | |
| Second level of struts | 12 struts, Φ1200, thickness = 20 mm, pitch = 5 m | 2.5E+07 | |
| Third level of struts | 12 struts, Φ800, thickness = 25 mm, pitch = 5 m | 2.1E+08 | |
| Fourth level of struts | 12 struts, Φ800, thickness = 20 mm, pitch = 5 m | 1.8E+09 | |
| r. c. slab | Thickness ranging from 2.5 m to 5.5 m | |
| r. c. walls | Thickness = 2 m | |

| Table 2 |
| Construction stages/modeling phases |
| 1. Construction of T-shaped reinforced concrete slurry walls |
| 2. Excavation (4 m b.g.s.) and dewatering from the bottom of the excavation |
| 3. Realization of reinforced concrete slab |
| 4. Excavation (11 m b.g.s.) and dewatering from the bottom of the excavation |
| 5. Installation of first level of struts (10.5 m b.g.s.) |
| 6. Excavation (15 m b.g.s.) and dewatering from the bottom of the excavation |
| 7. Installation of second level of struts (14.5 m b.g.s.) |
| 8. Excavation (19.5 m b.g.s.) and dewatering from the bottom of the excavation |
| 9. Installation of third level of struts (19 m b.g.s.) |
| 10. Excavation (23.5 m b.g.s.) and dewatering from the bottom of the excavation |
| 11. Installation of fourth level of struts (23 m b.g.s.) |
| 12. Excavation (28 m b.g.s.) and dewatering from the bottom of the excavation |
| 13. Removal of struts and construction of reinforced concrete walls |
| 14. End of dewatering |

| Table 3 |
| Geotechnical soil properties. |
| | Loose sand | Pyroclastic sand | Tuff (bedrock) |
| γ_s (kN/m^3) | 14 | 14 | 14 |
| γ_s sat (kN/m^3) | 16 | 16 | 16 |
| k_o = k_c (m/s) | 10^-5 | 10^-5 | 10^-5–10^-6 |
| E_j0 (kN/m^3) | 15,000 | 15,000 | 1.0E+06 |
| C_y (kN/m^3) | 5 | 0 | 500 |
| φ_d | 36 | 36 | 27 |
| k_o | 0.5 | 0.5 | 0.5 |

| Table 4 |
| H-S input parameters. |
| Parameter | Explanation | Conventional estimate | Model values |
| φ | Friction angle | Slope of failure line in σ_v–σ_s stress space | 36 |
| c | Cohesion | y- Axis intercept in σ_v–σ_s stress space | 0 |
| ψ | Dilatancy angle | Function of ψ_p and ψ_u | 0° |
| E_ref | Tangent stiffness for primary oedometer loading | y- Axis intercept in log(σ_1/σ_v)-log(Eref) space | 35,000 KPa |
| E_00 | Secant stiffness in standard drained triaxial test | y- Axis intercept in log(σ_3/σ_v)-log(E_00) space | 50,000 KPa |
| m | Power for stress-level dependency of stiffness | Slope of trendline in log(σ_3/σ_v)-log(E_00) space | 0.5 |
| Advanced parameters | | | |
| σ_0 | Unloading–reloading stiffness | Default = σ_0 | |
| R_f | Failure ratio q/q_u | Default = 0.9 | 0.9 |
| ν_s | Poisson’s ratio | Default = 0.2 | 0.2 |
| k_o | k_o value for normally consolidated soil conditions | Default = 1 – sin φ | 1 – sin φ |
supported by two thick reinforced concrete slabs, respectively at the top and bottom of the open pit, and four levels of temporary struts later substituted by permanent reinforced-concrete walls. The structural characteristics of the support system are presented in Table 1. Sixteen construction stages are needed to perform the excavation works (Table 2). An equal number of modeling phases is employed to simulate all these construction activities in a plane-strain numerical analysis, employing a mesh with 15-node triangular elements, of a significant cross section through the excavation. The 3-layer simplified soil stratigraphy, i.e. a loose sand upper soil layer overlying a pyroclastic sandy soil layer (locally named pozzolana) and a bottom tuff bedrock, and the mechanical properties of these layers derive from a geotechnical characterization at the site carried out using the results from a number of continuous core drilling borings, in situ penetrometer tests and laboratory identification, oedometric and direct shear tests (Table 3).

The numerical analysis of the case study is conducted using the Finite Element Method (FEM) implemented in the geotechnical commercial code PLAXIS (developed by Plaxis bv). The presence of buildings on the right side of the excavation is accounted for applying an uniformly distributed pressure over the ground surface (150 kPa) starting from a distance of 16.5 m from the excavation edge. The vertical drains were modeled with soil cluster elements characterized by relatively low deformability, relatively high strength and a permeability coefficient equal to 0.001 m/s.

The reinforced concrete slabs and walls were modeled with elastic elements characterized by relatively low deformability, relatively high strength and a permeability coefficient equal to 0.001 m/s.

The other structural element were modeled as elastic beams (see Table 1 for the values of axial and bending stiffness employed). Given the values of the hydraulic conductivities of all soil layers, the analysis has been carried out in drained conditions at each phase, thus assuming that the duration of each construction phase is long enough to ensure the complete dissipation of the excavation-induced changes in groundwater pressure before the beginning of the next phase.

In a finite element simulation of a geotechnical problem, the calibration of the models used to reproduce the soil behavior often poses significant challenges as real soil is a highly nonlinear material, with both strength and stiffness depending on stress and strain levels. A single constitutive model that can describe every aspect of the soil behavior does not exist, yet many constitutive models have been developed that can capture many of the important features of the soil behavior. For the case study considered herein, the excavation-induced strains within the soil volume affected by stress changes, which are characterized by a complex mixture of extension and compression stress paths in different areas of that volume, mostly involve the thick pyroclastic sandy soil layer. Therefore, this layer requires the use of a elasto-plastic constitutive model assuming yield criteria associated to both shear and volumetric strains. In PLAXIS, such characteristics are implemented within the hardening-soil H-S model [40], which is based on a Mohr–Coulomb strength criterion and two families of yielding surfaces: the ‘yield cap surface’, used to compute the distortional plastic strains with a non-associated flow rule and with a plastic potential defined to ensure a hyperbolic response upon loading in axisymmetric conditions. The parameters of the H-S model are: three strength parameters \( (\phi', c', R_f) \), five stiffness parameters \( (E_{ref}^x, E_{ref}^y, E_{ref}^z, v_{ref}, m) \), the dilatancy angle \( (\Psi) \) and the coefficient of earth pressure at rest \( (k_0) \). Table 4 shows their meaning, the conventional way of estimating them and the values for the numerical simulation of the case study. Regarding the other two soil layers, the loose sand upper soil layer and the bottom tuff bedrock, they may be adequately modeled using the relatively simple elasto-perfectly plastic Mohr–Coulomb MC model, which employs the following five input parameters: two stiffness parameters \( (E, v) \), two strength parameters \( (\phi', c') \), the dilatancy angle \( (\Psi) \). The values of parame-
ters $E$, $\phi'$ and $C'$ are reported in Table 3. The values of parameters $V$ and $\Psi$, assumed equal for both strata, are set to 0.2 and 0 respectively.

On the basis of information derived from the design of the excavation system, a parametric analysis has been conducted, employing four numerical simulations of the model (Table 5), to study the effect, on the computed results, of the following two significant model variables: the depth of the boundary between the pyroclastic and tuff layers, $E_{\text{lev}}$; the relative permeability between the two soils and the tuff layers, $k$. Regarding the depth of the boundary between the pyroclastic and tuff layers, $E_{\text{lev}}$, which is assumed to be horizontal, two cases are considered to account for the uncertainty related to the stratigraphy in the area, with minimum and maximum estimated depths respectively equal to 34 m and 46 m. Fig. 5 shows, as an example, the mesh adopted for the first case (i.e. simulations 1 and 2). Regarding the relative permeability of the three layers, $k$, two cases are considered to account for the uncertainty related to the estimation of their hydraulic conductivities. In particular, they are either assumed to be equal (homogeneous) or a difference of one order of magnitude (heterogeneous) is assumed between more permeable soil layers and a less permeable underlying rock stratum.

The results of the parametric analysis for the 15th excavation stage, i.e. the construction stage related to the maximum values of ground settlements before a final stage of swelling due to the end of dewatering, are shown in Fig. 6. The results of the four simulations are reported in two charts where the ground movements, respectively vertical (Fig. 6a) and horizontal (Fig. 6b), are plotted against the distance from the excavation edge, both in dimensional

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\mu$</th>
<th>$Cv$ (%)</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$: $-\log (\log (m/s))$</td>
<td>5</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>$B$: $E_{\text{ref}}$ (kPa)</td>
<td>50,000</td>
<td>30</td>
<td>15,000</td>
</tr>
<tr>
<td>$C$: $\phi'$</td>
<td>36</td>
<td>10</td>
<td>3.6</td>
</tr>
</tbody>
</table>

Combination for PEM analysis: $\mu \pm \sigma$. $A' = 6$; $A' = 4$; $B' = 65,000$ kPa; $B' = 55,000$ kPa; $C' = 32.4$; $C' = 39.6$. 

**Fig. 6.** Results of the parametric analysis for the 15th excavation stage: vertical (a) and horizontal (b) ground displacements.

**Table 6**

First and second moments of the relevant input random variables.
and dimensionless format. Regarding the vertical settlements, all the simulations indicate a concave profile of the settlement trough close to the excavation (i.e. ‘sagging’ zone) followed by a convex shape (i.e. ‘hogging’ zone) starting from a certain distance from the excavation edge. However, the position of both the point of maximum displacements and the point of inflection of the profile are different in the various simulations. A comparison with the settlement troughs predicted by the empirical methods also shows that the predicted maximum displacements are higher than the corresponding ones computed close to the excavation edge and lower than the ones computed far from the excavation. Similar differences among the four simulations also involve the computed horizontal settlements, whose maximum values increase and move away from the excavation edge as the thickness of the pyroclastic

Fig. 7. Results of the parametric analysis: vertical (a) and horizontal (b) displacements 20 m away from the excavation edge vs excavation stages.

Fig. 8. Results of the PEM analysis for the 15th excavation stage: vertical ground displacement profiles.
In both cases, simulation No. 3 is the most critical. The development of the computed ground movements during the construction stages is reported in Fig. 7, with reference to a point 20 m away from the excavation edge. The vertical settlements indicate that the effect of both model variables is comparable and starts being significant from the second excavation step, i.e. from phase 5. Regarding the horizontal movements, only the thickness of the pyroclastic layer significantly contributes to differentiate the four simulations and the differences only occur for the final excavation steps, i.e. from phase 11. On the basis of these results, and with reference to the procedure described in Fig. 2, the model which will be used in the probabilistic evaluation of the computed ground displacements around the excavation system is simulation No. 3, i.e. the simulation producing the most critical results with reference to both vertical and horizontal movements.

4.2. Probabilistic analysis of settlements

As described in Fig. 2, the probabilistic analysis of the computed ground displacements around the excavation system and the subsequent evaluation of the exceeding probability of damage to the affected buildings requires the choice and the estimation of a num-

![Fig. 9. Results of the PEM analysis for the 15th excavation stage: horizontal ground displacement profiles.](image)

![Fig. 10. Results of the PEM analysis for the 15th excavation stage: rotations of the vertical ground displacement profiles.](image)
ber of relevant independent geotechnical parameters, i.e. the random variables of the prediction model. Based on previous experience with the use of the H-S constitutive law to model the behavior of soil layers affected by excavation-induced deformations [41,42] two out of the six basic H-S model parameters are included in the probabilistic analysis, as they may be considered both relevant and uncorrelated for the numerical simulation of the case study: the reference stiffness modulus, \( E_{\text{ref}} \), and the friction angle, \( \phi' \). A third important input parameter considered as a random variable of the model is the base-10 logarithm of the coefficient of hydraulic conductivity of the strata, \( \log (k) \), as its variation also affects the final computed deformations. Table 6 shows the mean values, the coefficients of variation and the standard deviations of these parameters. The estimates of the mean values are the same used in the parametric analysis, the estimates of their second moments are based on the suggestions provided by Lumb [37].

The Point Estimate Method [35,36] is used for estimating the probability density functions of the relevant model results. As the input random variables are three, the total number of simulations to be run for the PEM analysis is 8. For each random variable of the analysis, the values of the mean, \( \mu \), plus or minus one standard deviation, \( \sigma \), are considered (Table 6). The results of all the

Fig. 11. Results of the PEM analysis for the 15th excavation stage: curvature of the vertical ground displacement profiles.

Fig. 12. Log-normal probability density function of the vertical ground displacements at \( x/H = 0.714 \).
Simulations are shown in Figs. 8–11. In particular, Figs. 8 and 9 show the vertical and horizontal ground displacements at different distances from the excavation edge; while Figs. 10 and 11 show the rotations and curvatures, respectively, related to the vertical displacements. These latter magnitudes are computed analytically, respectively as the first and second derivative, of the continuous function of the vertical displacements with distance, which are in turn defined, as shown in Fig. 8, by interpolating the vertical displacements computed every 10 m. As expected, the results show sagging deformations close to the excavation and hogging deformations after a distance of about 1.1–1.3x/H.

Assuming that the input parameters are independent and uncorrelated, the results of the eight simulations have the same probability of occurrence and can be used to compute the probability density functions of every other deformation variable correlated to them by using the maximum likelihood technique. In this case, log-normal density functions of the vertical and horizontal displacements are assumed after computing the first and second moment of the distribution from the simulation results at every distance from the excavation edge. The log-normal monovariate and multivariate density functions are computed according to the following equations:
\[ f(x) = \frac{1}{2\pi\sigma x} \exp \left\{ -\frac{1}{2} \left( \frac{\log x - \mu}{\sigma} \right)^2 \right\} \]

where \( x \) is the value of the considered variable (e.g. vertical or horizontal displacement); \( \mu \) and \( \sigma \) are the values of the first and second moments of that variable.

\[ f(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2 x_1 x_2} \exp \left\{ -\frac{1}{2} \left( \frac{\log x_1 - \mu_1}{\sigma_1} \right)^2 + \left( \frac{\log x_2 - \mu_2}{\sigma_2} \right)^2 \right\} \]

where \( x_1 \) and \( x_2 \) are the values of the considered variables (e.g. \( \Delta/L \) and \( \delta_p \) for the Burland functions [12]); \( \mu_1, \mu_2, \sigma_1 \) and \( \sigma_2 \) are the values of the first and second moments of the two variables.

**4.3. Probabilistic analysis of damage to buildings**

The reliability analysis of the excavation-induced damage to the adjacent buildings has been conducted with reference to both reinforced concrete structures and masonry buildings. Following the two categories of damage criteria previously identified, the probabilistic analyses have been conducted in the first case with reference to a single deformation parameters, i.e. the damage indices \([10,13,27]\), and in the second case with reference to two deformation parameters, i.e. the two-parameter damage functions [12]. In the two cases the exceeding probability has been evaluated.
through the single or double integration of the log-normal density functions, respectively expressed by Eqs. (1) and (2).

Regarding the damage to reinforced concrete structures which are potentially affected by the excavation-induced ground settlement trough, the exceeding probability for two of the abovementioned damage indices, the vertical displacements and relative rotation limits [10,13,27], was evaluated under the hypothesis of variable distances between the building’s edge and the excavation edge. Fig. 13 shows the cumulative probability functions of the vertical settlements as a function of the distance from the edge of the excavation. As expected, the greater the distance, the lower the probability of occurrence with respect to a pre-assigned damage level. Similarly, Fig. 14 shows the exceeding probability of different relative rotation limits, in the case of buildings with pad foundations or strip footings with beam span equal to 5 m, with reference both to serviceability and ultimate limit states. The results of the analysis show that, if the construction distance from excavation edge is less than its depth, the exceeding probability for the building to suffer heavy and moderate damage is unacceptably high, both in terms of absolute settlements and relative rotations. For this latter the Eurocode 0 [43] reference Serviceability Limit State, SLS, and Ultimate Limit State, ULS, probabilities are also reported in Fig. 14. The computed probabilities, however, drastically reduce when the distance from the excavation edge is higher than two times the excavation depth.

As for the excavation-induced damage to masonry structures with load bearing walls, log-normal bivariate probability density functions have been evaluated by considering two deformation parameters as independent and uncorrelated variables. To this aim, the damage functions proposed by Burland [12] have been used and, thus, the two parameters are the horizontal strain \( \varepsilon_H = -\varepsilon_x \) and the deflection ratio \( \Delta/L \). The bivariate probabilistic analyses have been carried out, considering a 20 m tall \( (h) \) and 30 m wide \( (L) \) typical building, for different distances between the building and the excavation edges. Fig. 15 shows the contour lines of the log-normal bivariate probability density functions compared to the damage functions proposed by Burland [12], plotted in the case of buildings located at \( x/H = 0.714 \) and \( x/H = 1.25 \) from the excavation edge. It is important to underline that the considered damage criteria only apply in the hogging zone, yet their use is justified by the fact that most of the building lies, in the considered cases, within this zone. Fig. 16 shows the comparison between the exceeding probability corresponding to different damage levels, as a function of the building–excavation distance, and the Eurocode 0 [43] reference SLS and ULS probabilities. The results show that the exceeding probability of severe damage level is higher than the one computed for the reinforced concrete structures. However, such probability quickly reduces when the building–excavation distance becomes greater than 1.5 times the excavation depth. As for the moderate and light damage levels, they present a slightly increasing trend with the building–excavation distance. This behavior is mainly due to fact that the horizontal strain deformations are greater in the hogging zone than in the sagging zone and therefore critical regions of the limit domain may occur, as already stated in other studies [11,12,21,44], at large distances from the excavation edge.

5. Concluding remarks

The probabilistic methodology presented in this paper has the aim to estimate excavation-induced damages to existing structures within the context of a reliability-based design approach based on monovariate or multivariate probabilistic analyses. At first, ground displacements induced by deep excavation systems are evaluated as a function of the probabilistic characterization of the model input parameters which are most relevant for the system response. Then, the damage induced on existing structures affected by the excavation-induced displacements is evaluated considering: the typology of the structures, appropriately defined damage functions and thresholds; the relative position between the excavation and the structures. Finally, exceeding probabilities associated to relevant potential damage states of the existing structures are computed. The procedure can be applied to every structural typology for which a significant damage function is defined. The methodology is meant to be used to evaluate, within a probabilistic framework and during the design phase of an excavation system, potential adverse effects on the urbanized area around the excavation.

The proposed methodology has been applied to a case-study: the design of an excavation system part of a new subway station.
in the city of Naples (Italy). The probabilistic analysis of the excavation-induced damage to the adjacent buildings has been conducted with reference to two typologies of buildings: reinforced concrete structures and masonry buildings. In the case of reinforced concrete structures, the exceeding probability was computed using a monovariate analysis based on a damage index associated with either vertical displacements or relative rotations thresholds. Whereas, for the excavation-induced damage to masonry structures, the probability of damage was computed on the basis of bivariate probability density functions evaluated by considering two deformation parameters as independent and uncorrelated variables. To this aim, the damage functions proposed by Burland [12] have been used and, thus, the two deformation parameters used are, respectively: the horizontal strain and the deflection ratio.

References