From the Observational Method to “Observational Modelling” of Geotechnical Engineering Boundary Value Problems

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Abstract: The observational method is a framework wherein construction and design procedures and details of a geotechnical engineering project are adjusted based upon observations and measurements made as construction proceeds. The term “observational modelling” is herein used to indicate methods and procedures that use inverse analysis techniques to update the numerical model of a boundary value problem using monitoring data. The article describes the main elements and characteristics of the proposed observational modeling approach. Moreover, it presents the effective application of such an approach to predict the soil displacements related to two different geotechnical boundary value problems: a deep excavation in urban environment and a slow-moving active landslide.

INTRODUCTION

For many geotechnical engineering projects, especially in urban environments, a monitoring program is often used to record, during construction, the important variables of the boundary value problem at hand. Monitoring data can be used to evaluate how well the actual construction process is proceeding in relation to the predicted behavior as well as to control the construction process and update the design of the project at early stages of constructions. The procedure to accomplish the latter task is usually referred to as the observational method (Peck 1969). Employing observed data in a timely enough fashion to be of practical use in a typical project is often a difficult task. The paper presents an approach which combines the observational method and inverse analysis techniques to update the model predictions of a geotechnical boundary value problem. In the second part of the article two case studies are used to demonstrate the applicability of the proposed approach.

THE OBSERVATIONAL METHOD AND INVERSE ANALYSIS

The observational method is a framework wherein construction and design procedures and details of a geotechnical engineering project are adjusted based upon observations and measurements made as construction proceeds (Peck 1969). Independently from the geotechnical boundary value problem at hand, an effective application of the observational method (Figure 1a) requires: a properly planned monitoring strategy—i.e., appropriate choice of variables to monitor, reliable monitoring data, criteria to evaluate the monitoring results; real–time analysis of the observations; alternative construction strategies to be adopted depending on the results of the data analysis. The main conceptual task related to the application of this framework is the continuous verification and updating of the design predictions as new field observations become available. If the geotechnical design strategy is based on the results of a model of the boundary value problem, this means being engaged in continuous model recalibration by (quasi real–time) back analysis.
Inverse analysis techniques may be very helpful in such an effort, as model calibration is performed by iteratively changing the estimates of its input parameters until the value of an objective function, which quantifies the errors between observed data and computed results, is minimized. When this occurs an “observational modelling” approach is in fact employed (Figure 1b). A possible definition for such an approach is the following: methods and procedures that use inverse analysis techniques to update, with time, the model of a boundary value problem using available monitoring data.

Figure 1. Schematics of: a) observational method; b) “observational modelling” approach for updating the design predictions of geotechnical boundary value problems

Important issues to deal with, when applying an observational modelling approach to predict the behaviour of time–dependent boundary value problems, are: modelling phases; accuracy of model predictions; parameterization of the observational model. Figure 2 shows a representation of the three time–dependent phases of an observational model: i) calibration, ii) validation, iii) prediction. To exemplify the idea of the time–dependency of the model predictions, two different times of analysis are considered. As shown, the future behaviour of the geotechnical boundary value problem predicted by the observational model changes with time, as it depends on the results of the calibration and validation phases and on the scenarios considered for the future conditions imposed on the boundary value problem.

Model uncertainty is another important issue to address. Uncertainties may be classified in two typological classes: i) aleatory uncertainty, which is due to the natural randomness of a variable; ii) epistemic uncertainty, which accounts for the lack of knowledge of a variable. The latter type includes measurement uncertainty (i.e. measurement errors), statistical uncertainty (due to limited information) and model uncertainty. Nadim (2002) defines the model uncertainty as a measure of the level of uncertainty about the bias value of the analysis method. Given this definition, uncertainties related to model predictions do not depend only on model uncertainty but also on a number of other factors. Figure 3 shows the relationship among the main factors affecting the accuracy of model predictions. Soil investigation activities, measurements errors and future scenarios are indeed related to model input factors such as: the boundary value problem schematization within the model (e.g., geometry, stratigraphy, soil constitutive laws); the estimates of the model input parameters; the model initial and boundary conditions. The uncertainties arising from the assumptions needed to define these factors contribute, together with the model uncertainty, to the accuracy of the model predictions.

Adequate parameterization of the observational model is also key to develop a “well–posed” problem. A well-posed inverse analysis problem is an optimization scheme which is able to effectively minimize the objective function of a simulation while yielding reasonable estimates of
the input parameters optimized. Inverse analysis algorithms allow the simultaneous calibration of multiple input parameters. However, in most practical problems, it is not possible to use these algorithms to estimate all the input parameter of a model. The number and type of input parameters that one can expect to estimate simultaneously depend upon many factors, including: the characteristics of the soil constitutive laws; how the model parameters are used by the model (e.g., within the element stiffness matrix in a finite element formulation); site stratigraphy; number and type of observations available; characteristics of the simulated system; computational time issues. To this aim, Calvello and Finno (2004) proposed a three–step procedure which uses the results of sensitivity analyses conducted on the soil model parameters for the identification of the relevant and uncorrelated parameters to calibrate. Initially, the number of relevant and uncorrelated parameters of the constitutive model chosen to simulate the soil behaviour is determined. This number depends upon the characteristics of the model, the type of observations available and the stress conditions in the soil. Secondly, the soil layers and the type of soil model used to simulate the layers are considered. An additional sensitivity analysis may be necessary, within this step, to check for correlations between parameters relative to different layers. The total number of relevant parameters of the boundary value problem is then determined. Finally, a third step is needed for a further reduction on the number of parameters to optimize simultaneously. This reduction depends on the total number of observations available at each optimizations stage and on computational time issues. Figure 4 shows a conceptual flowchart depicting the use of this procedure for the identification of the soil model parameters to optimize by inverse analysis.

Figure 5 visually summarizes the relationships among the main elements needed to effectively employ an observational modelling approach to update the model of a boundary value problem. The fundamental ingredients of an observational model are: the numerical model, the monitoring data, the inverse analysis algorithm, the analysis of the uncertainties. These four elements are interlinked within the boundary value problem space. The detailed specification of the characteristics of the both the elements and their connections necessarily requires sound engineering judgement. Indeed, starting with the model definition up to the assessment of the results of the analysis, the importance of the geotechnical engineer’s capabilities, knowledge and experience must never be underestimated. Engineering judgement can thus be defined as a sort of fifth fundamental ingredient of the observational modelling approach, perhaps the most important one.

Figure 2. Phases of an observational model: calibration, validation, prediction.
Figure 3. Factors affecting the accuracy of model predictions: uncertainty propagation paths.

Figure 4. Parameterization of the observational model of a geotechnical boundary value problem (modified after Calvello and Finno 2004).

Figure 5. Relationships among the fundamental ingredients of an observational model.
OBSERVATIONAL MODELLING CASE STUDIES

This section shows how the observational modelling approach can be affectively applied to predict the future behaviour of soil displacements for two very different geotechnical boundary value problems: a deep excavation in urban environment and a slow-moving active landslide. In the first case study, the deformations of the soil surrounding an excavation are recorded by inclinometers, which measure horizontal displacements at various depths at discrete locations around the construction site. Within the inverse analysis procedure, the measured displacements are used to update the predictions of the final movements around the excavation from data recorded at early stages of construction. In the second case study, numerous piezometers and inclinometers monitor both the groundwater regime and the kinematic behaviour of a slow-moving active landslide in clayey soils. The approach is used, in this case, to calibrate the main parameters of a numerical procedure relating the landslide movements to rainfall.

A versatile inverse analysis algorithm: UCODE

The inverse analysis algorithm employed to calibrate the numerical model of the boundary value problems reported herein is, in both cases, UCODE (Poeter and Hill 1999). UCODE is a computer code designed to allow inverse modelling posed as a parameter estimation problem. UCODE can be effectively used in geotechnical modelling because it works with any application software that can be executed in batch mode. Its model–independency allows the chosen numerical code to be used as a “closed box” in which modifications only involve model input values. Figure 6 shows a flowchart of the parameter optimization algorithm used in UCODE.

Figure 6. Flowchart of the parameter optimization procedure employed by the UCODE algorithm (modified after Finno and Calvello 2005)
The weighted least–squares objective function \( S(b) \) is expressed by:

\[
S(b) = \left( y - y'(b) \right)^T \omega \left( y - y'(b) \right) = e^T \omega e \tag{1}
\]

where: \( b \) is a vector containing values of the number of parameters to be estimated; \( y \) is the vector of the observations being matched by the regression; \( y'(b) \) is the vector of the computed values which correspond to observations; \( \omega \) is the weight matrix; \( e \) is the vector of residuals.

Non–linear regression is an iterative process. The modified Gauss–Newton method used to update the input parameters is expressed as:

\[
\left( C^T X_r^T \omega X_r C + m_r I \right) C^{-1} d_r = C^T X_r^T \omega \left( y - y'(b_r) \right) \tag{2}
\]

\[
b_{r+1} = \rho_r d_r + b_r \tag{3}
\]

where: \( d_r \) is the vector used to update the parameter estimates \( b_r \); \( r \) is the parameter estimation iteration number; \( X_r \) is the sensitivity matrix \( (X_{ij} = \partial y_i / \partial b_j) \) evaluated at parameter estimate \( b_r \); \( C \) is a diagonal scaling matrix with elements \( c_{jj} \) equal to \( 1/\sqrt{(X^T \omega X)_{jj}} \); \( I \) is the identity matrix; \( m_r \) is the Marquardt parameter (Marquardt 1963) used to improve regression performance; \( \rho_r \) is a damping parameter. Initially, the Marquardt parameter is set equal to 0; for iterations in which the vector \( d_r \) defines parameter changes that are unlikely to reduce the value of the objective function, as determined by the Cooley and Naff (1990) condition, \( m_r \) is increased by \( 1.5m_{r(old)} + 0.001 \) until the condition is no longer met.

Multiple runs of the model are required to update the input parameters at a given iteration because the sensitivity matrix \( X_r \) is computed using a perturbation method. At any iteration every input parameter \( b_r \) is independently perturbed by a fractional amount to compute the results’ response to its change. At any given iteration, after performing the modified Gauss–Newton optimization (Eq. 2 and 3), UCODE decides whether the updated model is optimized according to two convergence criteria. The parameter estimation is said to converge if either: i) the maximum parameter change of a given iteration is less than a user–defined percentage of the value of the parameter at the previous iteration; ii) the objective function, \( S(b) \), changes less than a user–defined amount for three consecutive iterations. When the model is optimized the final set of input parameters is used to run the model one last time and produce final updated results.

Different quantities can be used to evaluate the final model fit, among which: the value of the objective function; weighted residuals plotted on maps or time graphs; weighted observations plotted against weighted simulated values; the model error variance, i.e. an indicator of the overall magnitude of the weighted residuals. The relative importance of the input parameters being simultaneously estimated can be derived by parameter statistics, such as: the sensitivity of the predictions to changes in parameters values, the variance–covariance matrix, confidence intervals and coefficients of variation.

**Supported Excavation Case Study**

This case study shows how inverse analysis techniques may be used to calibrate the finite element model of a deep excavation in an urban setting (Calvello and Finno 2003; Calvello and Finno 2004; Finno and Calvello 2005). The excavation consisted of removing about 12 m of soil within 2 m of a school supported on shallow foundations. The support system consisted of secant pile walls, one level of cross–lot bracing and two levels of tie–backs. Ground movements during construction were recorded using inclinometers installed around the excavation site.

The commercial software PLAXIS 7.11 was used to model the response of the soil around the excavation in plane–strain conditions (Figure 7). The soil stratigraphy was assumed to be uniform across the site. Eight soil layers were modelled: a fill layer overlaying a clay crust, a
compressible clay deposit consisting of four distinct clay layers, and a relatively incompressible deposit consisting of two clay layers. The fill layer was modelled as an elastic–perfectly plastic Mohr–Coulomb material, whereas all clays layers were modelled using the H–S model (Schanz et al. 1999). Figure 8 shows the soil profile, a schematic of the support system and the observation points retrieved from the inclinometer data for the five construction stages for which the model predictions are updated. Observations from two inclinometers on opposite sides of the excavation were used to compare the computed displacements with the field data. The observations—13 for the east side and 11 for the west—were computed at the intersection between the finite element mesh and the inclinometer location. The observations for the last two stages on the west side are not reported because the inclinometer was destroyed by construction activities after Stage 3. Table 1 shows the initial values of the six basic H–S input parameters for the five clay layers calibrated by inverse analysis. The parameters are: friction angle, \( \phi \); cohesion, \( c \); dilation angle, \( \psi \); reference secant Young’s modulus at 50% stress level, \( E_{50}^{ref} \); reference oedometer tangent modulus, \( E_{oed}^{ref} \); exponent \( m \) relating the reference moduli to the stress level dependent moduli \( E_{50}, E_{oed} \) and \( E_{ur} \) (i.e., the unloading modulus). The initial estimates of the input parameters for Layers 1 to 4 were based on triaxial test results; the initial values of the parameters for Layer 5, in the absence of data from investigations, were assigned in relation to the input parameters assigned to Layer 4—same values of strength parameters and 50% increase on the values of the two stiffness parameters.

The input parameters optimized by inverse analysis were chosen following the procedure described in Figure 4. The results of a sensitivity analysis performed on the H–S basic parameters indicated that the parameters that are most relevant to the excavation problem are \( E_{50}^{ref}, m \) and \( \phi \) (Calvello and Finno 2004). From a simulation perspective, results show that the parameters that most influence the simulation are the ones relative to layers 1, 3, and 4. Layer 1 is the softest soil layer, thus its major influence on the displacement results is expected. Layer 3 is the stratum wherein the excavation ends. Layer 4 is the stiff clay layer below the bottom of the excavation into which the secant pile wall is tipped. The high sensitivity values of this stratum indicate that the strength and the stiffness of the clay below the excavation have significant impact on movements. The sensitivity analysis also indicates that the observations relative to a soil layer are mainly influenced by changes in that soil layer’s parameters. Table 2 shows the correlation coefficients between the three most relevant parameters at every layer. The rather high correlation between \( E_{50}^{ref} \) and \( m \) indicate that these parameters are not likely to be simultaneously and uniquely optimized, even though the results of the analysis are sensitive to both.

Figure 7. Schematic of the excavation system model (source: Calvello and Finno 2004)
Figure 8. Scheme of retaining system and observations points used from inclinometer readings (source: Finno and Calvello 2005)

Table 1. Initial values of Hardening–Soil parameters for the five clay layers.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Layer 1</th>
<th>Layer 2</th>
<th>Layer 3</th>
<th>Layer 4</th>
<th>Layer 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>23.4</td>
<td>23.4</td>
<td>25.6</td>
<td>32.8</td>
<td>32.8</td>
</tr>
<tr>
<td>c (kPa)</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$E_{50}^{\text{ref}}$ (kPa)</td>
<td>226</td>
<td>288</td>
<td>288</td>
<td>413</td>
<td>619</td>
</tr>
<tr>
<td>$E_{vcl}^{\text{ref}}$ (kPa)</td>
<td>158</td>
<td>202</td>
<td>202</td>
<td>289</td>
<td>433</td>
</tr>
<tr>
<td>m</td>
<td>0.8</td>
<td>0.8</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Table 2. Correlation coefficients between $E_{50}^{\text{ref}}$, m and $\phi$ at every layer.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Between parameters</th>
<th>Value</th>
<th>Between parameters</th>
<th>Value</th>
<th>Between parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>-.70</td>
<td></td>
<td>-.42</td>
<td></td>
<td>.33</td>
</tr>
<tr>
<td>2</td>
<td>$E_{50}^{\text{ref}}$ and m</td>
<td>-.85</td>
<td>$E_{50}^{\text{ref}}$ and m</td>
<td>-.59</td>
<td>m and $\phi$</td>
<td>.41</td>
</tr>
<tr>
<td>3</td>
<td>m</td>
<td>-.87</td>
<td>$\phi$</td>
<td>-.58</td>
<td>m and $\phi$</td>
<td>.25</td>
</tr>
<tr>
<td>4</td>
<td>$\phi$</td>
<td>-.99</td>
<td>$\phi$</td>
<td>-.07</td>
<td></td>
<td>-.14</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>-.95</td>
<td></td>
<td>.39</td>
<td></td>
<td>-.56</td>
</tr>
</tbody>
</table>
For calibration purposes parameter $E_{50}^{ref}$, rather than parameter $m$, was chosen to “represent” the stiffness of the H–S model because changes in $E_{50}^{ref}$ values also produce changes in the values of parameters $E_{oed}^{ref}$ (equal to 0.7 times $E_{50}^{ref}$) and $E_{ur}^{ref}$ (equal to 0.7 times $E_{50}^{ref}$), thus its calibration can be considered as representative of the calibration of all H–S stiffness parameters. A further reduction of the parameters to optimize was also necessary to establish a well–posed problem. To this aim, layers 1 and 2 were combined because: layer 2 had a much lower impact on the computed results, as indicated by the low values of composite scaled sensitivities; the two layers are derived from the same geologic stratum. Moreover, the stiffness parameters ($E_{50}^{ref}$) were chosen over the failure parameters ($\phi$) because: the excavation–induced stress conditions in the soil around this excavation were, for the most part, far from failure; the laboratory estimated values of $\phi$ are judged to be more accurate than $E_{50}^{ref}$. When the stiffness and failure parameters were optimized simultaneously or only the failure parameters were calibrated, the regression analysis never converged to reasonable values. This emphasizes the point that convergence does not necessarily ensure that reasonable results are attained when optimizing a highly nonlinear boundary value problem. A summary of the inverse analysis setup is presented in Table 3. Fifteen parameters were updated at every iteration, but only three of them—$E_1 = E_{50}^{ref}$ (soil layer 1), $E_3 = E_{50}^{ref}$ (soil layer 3), $E_4 = E_{50}^{ref}$ (soil layer 4)—were directly estimated by the optimization algorithm. Note that changing the values of $E_{50}^{ref}$ is not the same as merely changing the elastic parameters of an elastoplastic or linear elastic soil model because the hardening soil responses are nonlinear below the cap and the stiffness depends on more than $E_{50}^{ref}$.

Table 3. Summary of the main choices for the inverse analysis setup.

<table>
<thead>
<tr>
<th>Geotechnical setup</th>
<th>Type of observations</th>
<th>readings from inclinometers (west, east)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of observations</td>
<td>11 readings per construction stage (west)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>13 readings per construction stage (east)</td>
<td></td>
</tr>
<tr>
<td>Input parameter optimized</td>
<td>H–S stiffness parameters of 5 soil layers: $E_{50}^{ref}$, $E_{oed}^{ref} = 0.7 E_{50}^{ref}$, $E_{ur}^{ref} = 3 E_{50}^{ref}$</td>
<td></td>
</tr>
<tr>
<td>Input parameter calibrated</td>
<td>$E_i = E_{50}^{ref}$ of soil layer i, for $i = 1$ to 5 assuming: $E_2 = E_1$; $E_5 = 1.5 E_4$</td>
<td></td>
</tr>
</tbody>
</table>

| Numerical setup | Observations' weighting | $\sigma^2$ = measurement error variance |
| | (see Finno and Calvello, 2005) |
| Convergence criteria | TOL = SOSR = 5% |
| Regression variables | MAX–CHANGE = 0.5 |
| Sensitivity calculation | PERTURBATION = 0.01 |

The simplest way to evaluate the difference between the results of the numerical simulations based on the initial estimates of the parameters and the optimized ones is to compare the inclinometer data with the computed horizontal displacements for the two cases. Figure 9 shows the visual fit between the observations and the results computed before (initial) and after the calibration by inverse analysis (best–fit). The comparison shows that the initial simulation computes displacements significantly larger than the measured ones at every construction stage (the maximum computed displacements at stage 5 are about two times the measured ones) and the computed displacement profiles result in significant and unrealistic movements in the lower clay layers. When the model is calibrated by inverse analysis, the fit between the computed and measured response is extremely good. At the end of the construction the maximum computed displacement exceeds the measured data by less than 10% and the distributions of lateral deformations are consistent throughout the excavation. The good fit shown in the Figure refers to
the final optimization, i.e. all observations (stages 1–5) were used to calibrate the finite element model of the excavation. Yet, the simulation was calibrated starting at stage 1 and recalibrated at every subsequent construction stage using the inclinometer data available up to that stage. The variation of the input parameters at the five optimization stages is shown in Figure 10 above a bar chart, representing the excavation depth normalized with respect to the excavation width. Results show that the maximum changes in parameter values occur at Stage 1, when the observations refer to the installation of the secant pile walls inducing movements throughout the compressible clay layers. A satisfactory calibration at this stage indicates that these movements were large enough to “exercise” the constitutive laws of all soil layers subsequently affected by the excavation. The results also indicate that the initial estimates of the stiffness parameters are significantly lower than the optimized values of the parameters. This trend could be expected because the initial values were based on results of triaxial compression tests on specimens taken from thin–wall tubes. Yet, if an analyst was to arbitrarily increase the initial stiffness parameters prior to optimization, the magnitude of the increase would be a matter of much judgment and, most likely, the parameter values would still require subsequent adjustments to provide good fits to the observed data. Portions of the increase in optimized stiffness between stages 2 and 3 may be a result of end effects of the excavation. The simulated excavation is really a three–dimensional problem modeled in plane strain. When the excavated depth is small, most of the wall can be adequately modeled as plane strain and, hence, little changes in parameters are noted between stage 1 and 2. As the excavation deepens, the ratio between excavation depth and excavation width increases and higher parameter values compensate for the lack of constraints in the out–of–plane direction.

![Figure 9](image)

Figure 9. Measured and computed horizontal displacements for initial and best–fit estimates of parameters (source: Calvello and Finno 2004)
Active Landslide Case Study

The case study refers to a well-monitored active slide in Central Italy (Bertini et al. 1984), characterized by very slow movements occurring within a narrow band of weathered bedrock overlaid by a clayey silt colluvial cover. The inverse modelling approach is herein used to calibrate the main parameters of a numerical procedure relating landslide movements along pre-existing slip surfaces to rainfall data (Calvello and Cascini 2006; Calvello et al. 2008). The procedure comprises: a transient seepage finite element analysis to compute the variations of pore water pressures due to rainfall; a limit equilibrium stability analysis to compute the factors of safety along the slip surface associated with transient pore pressure conditions; an empirical relationship between the factor of safety and the rate of displacement of the slide along the slip surface. The numerical procedure is divided in two parts: a groundwater model and a kinematic model (Figure 11). In the first part, monthly recorded rainfall data are used as time-dependent flow boundary conditions of the transient seepage analysis, while piezometric levels are used to calibrate the analysis by minimizing the errors between monitoring data and computed pore pressures. In the second part, measured inclinometric movements are used to calibrate the empirical relationship between the rate of displacement along the slip surface and the factor of safety, whose variation with time is computed by a time-dependent stability analysis.

The procedure is applied to an active landslide characterized by very slow movements (~cm/year) occurring within a narrow layer of soil. The monitoring data refer to a 4.5-year period (from 16/02/1980 to 23/06/1984) and include observations from one rain gauge, twelve piezometric cells and six inclinometers (Bertini et al. 1984, 1986). Such instruments were all installed along a section of the slope that can be considered representative of the entire mass movement developing on the left side of the valley (Figure 12). As shown in Figure 11, the finite element seepage analysis computes the transient pore pressure regime in the slope induced by the rainfall. The optimization algorithm minimizes the error between the measured piezometer levels and the numerically computed results by calibrating the hydraulic conductivity values of the three soil layers and one boundary condition. The commercial finite element code SEEP/W was used to compute the changes in pore pressures within the slope. The boundary conditions are: unit rate of flow monthly step function equal to the measured monthly rainfall data, $R(t)$, at the ground surface; impervious bottom boundary; constant head values on both the right, $H_{\text{right}}$, and left, $H_{\text{left}}$, boundaries. The hydraulic conductivities of all layers are assumed independent of the values of pore water pressure and, thus, they are only characterized by their saturated values. As
for the volumetric water content, a linear relationships with pore water pressure is adopted, whose slope is equal to the saturated oedometric compressibility, mw. For further details on the numerical analysis see Calvello et al. (2008).

Figure 11. Numerical procedure relating landslide movements along pre–existing slip surfaces to rainfall data (source: Calvello et al. 2008)

Figure 12. Cross section of landslide with location of installed instruments (source: Calvello et al. 2008)
With the above described hypotheses, the number of parameters characterizing each soil layer reduces to four: the hydraulic conductivity along the horizontal x–axis direction, \( k_x \); the anisotropy ratio \( k_r = k_y/k_x \); the anisotropy direction \( k_d \), which defines the direction of the x–axis with respect to the horizontal direction; the oedometric compressibility \( m_w \). Table 4 reports the initial and optimal values of the parameters used for the transient seepage analysis. Table 5 reports the variances of the 5 estimated parameters and the coefficients of variation of the calibrated values, the latter being a measure of the relative accuracy of the estimates. The initial values are mainly derived from the hydraulic properties of the soil layers reported by Bertini et al. (1986). Some of the calibrated values differ significantly from the initial estimates. In particular, the calibrated hydraulic conductivity of layer 2 (weathered bedrock), initially assumed to be equal to the hydraulic conductivity of the colluvium, although it was never measured on the field, is much smaller than its initial estimate and about one to two orders of magnitude smaller than the calibrated hydraulic conductivities of layers 1 and 3. Most probably, one would not have managed to pin–point this relevant hydraulic characteristic feature of the slope, which significantly influences the results of the numerical transient seepage analysis, without the joint calibration of the soil layers’ conductivities by inverse analysis.

Table 4. Main input parameters of seepage analysis: initial estimates (within brackets) and calibrated values (in bold if they differ from the initial estimates)

<table>
<thead>
<tr>
<th>SEEP parameter</th>
<th>Layer 1 colluvial cover</th>
<th>Layer 2 weathered bedrock</th>
<th>Layer 3 unweathered bedrock</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_x ) [m/day]</td>
<td>5</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>( k_r )</td>
<td>0.02</td>
<td>0.2</td>
<td>1</td>
</tr>
<tr>
<td>( k_d ) [°]</td>
<td>–10</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>( m_w ) [kPa⁻¹]</td>
<td>5.0E–05</td>
<td>2.0E–04</td>
<td>2.0E–04</td>
</tr>
<tr>
<td>( H_{left} ) [m a.s.l.]</td>
<td>175</td>
<td>176</td>
<td></td>
</tr>
</tbody>
</table>

Table 5. Variances and coefficients of variation of the seepage analysis calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( k_x ) (layer 1)</th>
<th>( k_r ) (layer 1)</th>
<th>( k_x ) (layer 2)</th>
<th>( k_x ) (layer 3)</th>
<th>( H_{left} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>calibrated value, ( \mu )</td>
<td>1 m/d</td>
<td>0.2</td>
<td>0.01 m/d</td>
<td>0.2 m/d</td>
<td>176 m</td>
</tr>
<tr>
<td>variance of estimate, ( \sigma^2 )</td>
<td>8.36E–02</td>
<td>9.86E–04</td>
<td>6.23E–06</td>
<td>1.23E–03</td>
<td>8.14E–02</td>
</tr>
<tr>
<td>CoV = ( \mu/\sigma )</td>
<td>28.9%</td>
<td>15.7%</td>
<td>25.0%</td>
<td>17.5%</td>
<td>0.2%</td>
</tr>
</tbody>
</table>

The kinematic model uses the pore water pressure variations computed by the groundwater model to predict the rate of movement along the slip surface. This is achieved, as shown in Figure 11, by combining a time–dependent stability analysis, whose results are expressed in terms of factors of safety, and an empirical relationship to convert the factors of safety in displacement rates. The limit equilibrium stability analysis is performed using the commercial code SLOPE/W. Within layer 2 (i.e. the weaker layer), multiple slip surfaces are defined, according to the displacement profiles recorded along inclinometers B and C. The results refer to the lowermost slip surface, located at the border between layers 1 and 2. The evolution with time of the factor of safety, \( F_s(t) \), is determined by running a number of simulations equal to the time steps defined in the transient seepage analysis (i.e. one time step every 15 days) and by using, at each time step, the related pore pressure distribution. The values of the input parameters of the analysis—i.e. unit weights of the soil layers and residual friction angle of layer 2—are not optimized but simply derived from what reported by Bertini et al. (1986). This assumption is justified by considering that the slip surface entirely develops within one soil stratum and, thus, the chosen value of the
friction angle influences only the computed safety factor but not its gradient with respect to time. The kinematic model assumes the existence of an empirical relationship between factor of safety and rate of displacement along the slip surface. The following two relations, defined by the input parameters $F_{\text{max}}$, $v_{\text{max}}$, $v_{\text{min}}$, and valid for $F \leq F_{\text{max}}$, are considered:

\[
v(t) = v_{\text{max}} \frac{F_{\text{max}} - F(t)}{F_{\text{max}} - 1} \tag{4}
\]

\[
v(t) = v_{\text{min}} 10^{(1 - \log F / \log F_{\text{max}}) \log v_{\text{max}} / v_{\text{min}}} \tag{5}
\]

Both expressions assume the existence of: (i) a threshold value of factor of safety ($F_{\text{max}}$) above which the displacement rate is null, and (ii) a maximum value of velocity ($v_{\text{max}}$) corresponding to a factor of safety of 1.0. Eq. 4 implies a linear relationship between the two variables, while Eq. 5 implies that the trend is linear in a log–log space below $F_{\text{max}}$, where the velocity is equal to $v_{\text{min}}$. Figure 13 shows the comparison, both for the initial and the optimized simulations, between the rates of displacement recorded during the first three years of monitoring (1980–1982) and those computed adopting the results obtained by the time dependent stability analysis. The values of the input parameters $F_{\text{max}}$, $v_{\text{max}}$, $v_{\text{min}}$ are also reported. The results clearly show the benefits of calibrating the kinematic model, as a significantly improved comparison between the numerical predictions and the recorded velocities is attained for both the adopted $F$–$v$ relationships. It is important to highlight that the model is calibrated using only a limited number of inclinometric observations, relative to about 8 months of measures and corresponding to the first surge of movements. This is done in order to use the rest of the observations to validate the performed calibration. In this case, the first surge of movements can be considered to be both significant for the model and representative of the future kinematic behaviour of the landslide and, thus, the inverse analysis problem proves well–posed.

Subsequently, the calibrated and validated models are used to predict the response of the slope to different rainfall scenarios. Two different times of analysis (see Fig. 14b) are considered: Time1=10/04/1983 (i.e. 1050 days after the beginning of the simulation) and Time2= 23/06/1984 (i.e. 1590 days after the beginning of the simulation). The first analysis does not use all the available measures, while the second one does. The two analyses at different times are used to evaluate the reliability of the considered rainfall scenarios by comparing the predicted velocities between Time1 and Time2 (analysis time=Time1) against the velocities in the same period computed using the recorded rainfall (analysis time=Time2). The rainfall scenarios used in the analysis are shown in Figure 14a. They refer to two stationary conditions, representing reasonable upper (T1-Rs-StMax) and lower bounds (T1-Rs-StMin), and a transient upper bound conservative distribution (T1-Rs-TrMax). The first two are computed using the maximum and the average recorded monthly rainfall. The last one is computed using, at each month, the maximum recorded monthly rainfall data relative to the same month of the year. Figure 14b shows the comparison between predicted displacement rates at analysis time Time1, using different rainfall scenarios, and the displacement rates of the calibrated and validated model at analysis time Time2, using the recorded rainfall (T2). For clarity, only the results relative to the linear relationship between factors of safety and rates of displacement are reported. The results show that, despite the simplicity of the considered rainfall scenarios, the upper and lower boundaries of the rates of displacement are properly identified. In particular, the maximum displacement rates predicted by the transient rainfall scenarios (T1-Rs-TrMax) well match the maximum displacement rates computed at Time2, when the recorded rainfall data are used (T2). However, at analysis time Time2 the model overestimates the observed velocities of the last surge of movements. If a recalibration of the model is performed (T2-recalib), the computed results better reproduce the latest surge of movements, while only slightly underestimating the previous ones.
Figure 13. Comparison between computed and measured velocities along the main slip surface: a) initial simulation; b) optimized simulation (modified after Calvello et al. 2008)

Figure 14. (a) Recorded rainfall and rainfall scenarios considered in the analyses; (b) Comparison among recorded and predicted velocities: at analysis time Time1 for the different rainfall scenarios; for the model validated at analysis time Time2; for the model recalibrated at analysis time Time2 (modified after Calvello and Cascini 2006)
CONCLUSIONS

The paper presented an approach, called “observational modelling,” which combines the observational method and inverse analysis techniques to update, with time, the model of a boundary value problem using available monitoring data. It has been shown how such an approach can be effectively used to predict the soil displacements related to geotechnical systems. To this aim, two different case studies have been presented, the first one dealing with the soil deformations induced by an excavation constructed close to existing facilities, the second one addressing the issue of modelling and forecasting rainfall–induced landslide displacements along existing slip surfaces. The first part of the paper, besides presenting the approach, highlighted the key role played by engineering judgment in defining the four main ingredients of an observational model, i.e. the numerical model, the monitoring data, the inverse analysis algorithm, the analysis of the uncertainties. Adequate engineering judgment is also required to properly consider the relationships among these ingredients. To this aim, the issues needing most consideration when employing an observational modelling approach are: 1) the definition of the calibration, validation and prediction modelling phases in relation to the available observations; 2) the identification of the factor affecting the accuracy of model predictions; 3) the parameterization of the inverse model, i.e. the identification of the model input parameters to optimize by inverse analysis. In the first case study, the used observations were soil horizontal displacements recorded by inclinometers at discrete locations around an excavation site. Within the inverse analysis procedure, the monitoring data were used to update the predictions of the final deformations around the excavation from data recorded at early stages of construction. The following main general conclusion can be drawn from the presented results: when dealing with a finite element simulation of a geotechnical project which involves the calibration of multiple soil layer, a good understanding of the boundary value problem is essential to define a well–posed inverse model.

In the second case study, the observations were retrieved from piezometric and inclinometric data monitoring both the groundwater regime and the kinematic behaviour of a slow–moving active landslide. The approach was used, in this case, to calibrate the main parameters of a numerical procedure relating the landslide movements to rainfall. The following general conclusion can be drawn from the comparison, at different analysis times, between recorded data and numerical results: as time passes and more monitoring data are available, a better understanding of the mechanisms behind the behavior of the slide is possible, thus allowing more reliable model predictions of the future displacements.

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REFERENCES


