Optimizing the MPM model of a reduced scale granular flow by inverse analysis

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ABSTRACT: MPM is a mesh-free method designed for large deformation problems. This paper presents the calibration by inverse analysis of a MPM model for a dry sand flow experiment conducted in a miniature flume. The simulations were carried out using the Anura3D code and adopting a Mohr-Coulomb frictional constitutive law for the propagating soil. An inverse analysis algorithm was used to evaluate the sensitivity and optimize some of the input parameters of the model. Four sets of observations have been considered to define the error functions, representing the soil thickness profiles along the slope at four distinct times. The results are presented in relation to: the ability of the model to simulate the behaviour of the sand flow; the sensitivity of the results to changes in the parameter values; the effectiveness of the calibration procedure; the type and number of observations used.

1 INTRODUCTION

Modelling the propagation stage of landslides is a relevant issue for urban planning and landslide risk mitigation (Fell et al., 2008) and the knowledge of the rheological properties of the propagating soil, i.e. the relationships between shear stress and shear strain rate, is a key ingredient of any physically-based model.

Rheology is difficult to assess experimentally because in-situ investigations provide information on soil characteristics before failure and laboratory equipment can hardly reproduce in-situ conditions for the propagation stage. Therefore, rheological properties are usually estimated through the back-analysis of case histories and best-fitting values are computed through trial-and-error procedures based on expert judgment and subjective evaluations.

In this paper an inverse analysis procedure is tested for a flume test of a dry granular flow, which can be considered a benchmark case for many phenomena occurring in the Alps. Previous contributions about similar flume tests and real landslides were proposed by Cascini et al. (2014), Cuomo et al. (2013, 2015) and Calvello et al. (2017) using Smooth Particle Hydrodynamics (SPH) models. Herein, numerical modelling is performed using the Material Point Method (MPM).

2 MATERIALS AND METHODS

2.1 Experimental test

Small scale experiments with flows of dry sand were carried out by Denlinger and Iverson (2001). The material propagated through a rectangular flume with a bed surface, inclined 31.4°, joined to a horizontal runout surface by a curved section with a 10 cm radius of curvature (Figure 1). A vertical gate spanning the entire flume width (20 cm) was positioned in the uppermost part of

![Figure 1. Schematic of the flume used for propagation tests of granular flows (Denlinger and Iverson, 2001).](https://bookshelf.vitalsource.com/#/books/9780429823190/cf/593/4/2@100:0.00)
the slope. About 290 cm$^3$ of loosely packed, well-sorted and well-rounded dry sand was placed behind the gate ensuring a horizontal soil surface.

The bulk density of the soil was approximated as 1600 kg/m$^3$. The flume bed was surfaced with Formica. The static bed frictional angles measured using tipping table tests of sand sliding across the Formica was reported as 29°±1.4°. The internal friction angle of the sand was reported equal to 40°±1°.

The experiment started by suddenly opening the entire gate. The flow accelerated, elongated, and thinned rapidly after the gate opened. A non-invasive optical shadowing technique was used to measure the soil thickness during the flow and at the end of the experiment.

When the leading edge of the flow reached the break in slope located 37.5 cm downslope from the gate, the sand that first reached the depositional area was only slightly pushed forward by subsequently arriving sand.

In the experiment considered herein, the sand deposition was complete 1.5 s after the flow release. Sand thicknesses normal to the flume bed are reported by Denlinger and Iverson (2001) at 0.32 s, 0.53 s, 0.93 s and 1.5 s after the gate released (Figure 2).

2.2 Material Point Method (MPM)

MPM is considered to be a variant of FEM, which utilizes structures the material points and the background mesh. All information (e.g. body forces, stresses, state variables,…) are stored in the material points and move through the background mesh during the simulation. The mesh is used only to solve the balance equations and to transfer information to the material points.

Since its adaptation for solid mechanic problems (Sulsky et al., 1994), a wide range of slope stability applications have used this method, such as: landslide and debris flow (Shin, 2009), wave breaking on a dike (Kafaji, 2013), levee failure propagation (Bandara and Soga, 2015), rainfall induced flow slide (Wang et al., 2016) and earthquake induced slope failure (Abe et al., 2017).

Here in, the MPM software ANURA 3D (http://www.mpm-dredge.eu/) has been used to model the flume test previously described. In order to simulate the contact between the sand and the basal surface, the contact formulation proposed by Bardenhagen et al. (2001) was employed. This algorithm is a predictor-corrector scheme, in which the velocity is predicted from the solution of each body separately and then corrected using the velocity of the two bodies following a Coulomb friction law (Kafaji, 2013). Thus, sliding of two adjacent surfaces depends on a parameter defined as frictional coefficient.

2.3 Inverse analysis

Inverse analysis works in the same way as a non-automated calibration approach: parameter values and other aspects of the model are adjusted until the model’s computed results match the observations made for the behaviour of the system. Herein, model calibration by inverse analysis is conducted using UCODE (Poeter and Hill 1998), a computer code designed to allow inverse modelling posed as a parameter estimation problem.

In UCODE the parameters are optimized by minimizing, using a modified Gauss-Newton method, the following weighted least-squares objective function $S(h)$:

$$ S(h) = \left[ y - y' (h)^T \right] \omega \left[ y - y' (h) \right] = e^T \omega e $$  

(1)
where: \( \mathbf{h} \) is the vector of the parameters being estimated; \( \mathbf{y} \) is the vector of the observations being matched by the regression; \( \mathbf{y}'(\mathbf{h}) \) is the vector of the corresponding computed values; \( \mathbf{w} \) is the weight matrix, being the weight of every observation taken as the inverse of its error variance; \( \mathbf{g} \) is the vector of residuals.

The objective function should be seen as a measure of the ability of the models to correctly represent the physical process. The following two convergence criteria are used at any given iteration: 1) maximum parameter change lower than a user-defined percentage of the value of the parameter at the previous iteration; ii) objective function changes lower than a user-defined amount for three consecutive iterations.

After the model is optimized, the final set of input parameters is used to run the numerical model one last time and produce the final “updated” results. More details on the inverse analysis procedure adopted herein can be found in Calvello (2014, 2017).

2.4 Observations

The longitudinal cross-sections of the propagating soil have been drawn, at different experimental times, using the contour lines presented in Figure 2. Figure 3 shows, as an example, the cross section highlighting the base of the apparatus and the position of the soil at the end of the test, corresponding to an experimental time equal to 1.5 s.

The adopted coordinate system employs a vertical axis starting at the level of the rightmost horizontal surface and a horizontal axis with the same longitudinal distances reported in Figure 2. The soil surface is discretized by means of 18 points, almost equally spaced along the horizontal axis.

The observations used in the inverse analysis of the MPM model are the values of the elevation of these points. When, at any given experimental time, the soil is not present at these locations, the elevation of the base of the flume is used.

3 CASE STUDY

3.1 MPM model

The MPM model was created adopting the Auran3D MPM code. The domain was discretized by 12'555 elements (Figure 4). The material points representing the soil are initially positioned in a relative small area located in the uppermost portion of the mesh comprising 440 elements. Each one of these elements initially contains 4 material points. The experimental gate was simulated by applying horizontal fixities at the right boundary of the soil domain. To initialize the soil stresses, a quasi-static calculation was carried out at the beginning of the simulation. Subsequently, the horizontal fixities were removed and the soil was allowed to propagate downwards along the slope.

An elastic perfectly plastic constitutive law is used to simulate the behavior of the soil. The constitutive model is based on the Mohr-Coulomb failure criterion and adopts 5 input parameters: stiffness modulus \( (E) \), Poisson’s ratio \( (\nu) \), cohesion \( (c) \), friction angle \( (\phi) \), and dilatancy angle \( (\psi) \). In addition to these, two other parameters are also needed to define the initial conditions of the soil: porosity \( (n) \), and specific gravity of the soil grains \( (G_s) \). The contact with the base of the experimental apparatus was simulated adopting a frictional law with a single input parameter: the contact coefficient \( (\mu) \).

The values of the input parameters of the initial numerical simulation were determined considering the values of the sand properties reported by Delinger and Iverson (2001) and the results of a numerical simulation already performed, for the same case study, by Ceccato and Simonini (2016).

They are equal to: \( E = 1000 \text{ kPa} \), \( \nu = 0.3 \), \( c = 0 \), \( \phi = 40^\circ \), \( \psi = 0 \), \( n = 0.4 \), \( G_s = 2.65 \).

![Figure 3](https://bookshelf.vitalsource.com/#/books/9780429823190/cfi/593/4/2) Observations, at time equal to 1.5 s, along the longitudinal cross-section of the flume used to calibrate the MPM model.

![Figure 4](https://bookshelf.vitalsource.com/#/books/9780429823190/cfi/593/4/2) Scheme of computational domain.
Considering the above conditions, the time needed to run one model simulation is approximately equal to 60%. The comparison between the experimental observations and the results of the initial MPM simulation is reported in Figure 5 considering the position of the soil at the end of the test, i.e. experimental time equal to 1.5 s. The numerical results of the MPM model are “stored”, at the end of each time step, at the location of the material points, which are of course moving within the mesh during the simulation. On the contrary, the 18 points used as observations for a given experimental time (Fig. 3) are fixed in space. A purposefully defined numerical algorithm is herein used to extract the values of the elevations of the MPM material points corresponding to the adopted observations. Buffer zones having a width equal to $D_b$ are defined at the location of each observation, i.e. longitudinal distance $X_i$, as follows:

$$X_{i,\text{buf}} = X_i - D_b$$ (2)
$$X_{i,\text{max}} = X_i + D_b$$ (3)

where: $X_{i,\text{buf}}$ is initial longitudinal distance of the buffer zone for the $i$-th observation; $X_{i,\text{max}}$ is final longitudinal distance of the buffer zone for the $i$-th observation.

As depicted in Figure 5, the numerical value to compare to the elevation of the $i$-th observation, at any given experimental time, is equal to the maximum elevation of all the material points falling within the corresponding buffer zone at that time.

### 3.2 Sensitivity analysis

The relative importance of the input parameters being simultaneously estimated by the adopted inverse analysis algorithm can be defined using: statistics representative of the sensitivity of the predictions to changes in parameters values; and statistics derived from the variance-covariance matrix.

Among the statistics able to evaluate the sensitivity of the predictions to parameters changes, the composite scaled sensitivities, $c_{ss,i}$, are herein used:

$$c_{ss,i} = \left[ \sum_{j=1}^{n} \left( \frac{\partial Y_j'}{\partial b_j} \right) b_j \rho_i^{1/2} N_D \right]^{1/2}$$

where: $Y_j'$ is the $i$-th simulated value; $X_i$ is the sensitivity of the $i$-th simulated value with respect to the $j$-th parameter; $b_j$ is the $j$-th estimated parameter; $\rho_i$ is the weight of the $i$-th observation, $N_D$ is the number of observations.

Multiple runs of the MPM model are required to compute the sensitivity matrix $(X_i)$. To this aim, a perturbation method is used. Every input parameter $b_j$ is independently perturbed by a fractional amount to compute the results’ response to its change. To this aim, all the available elevation data (72 observations) have been considered. Table 1 reports the values of the composite scaled sensitivities for 7 of the 8 input parameters of the MPM model. Parameter $c$ is not considered in the sensitivity analysis as its value is always assumed to be zero. The values of $c_{ss}$ were computed considering the input parameters values of the initial numerical simulation. The perturbations were always set to 1% of the parameter values. The results of the sensitivity analysis clearly indicate that the input parameter whose changes have the highest impact on the model results is the contact coefficient, $\mu$. Not surprisingly, among the 5 parameters used in the constitutive law adopted to simulate the soil behaviour, the highest composite scaled sensitivity value refers to the friction angle, $\phi$.

The results of the sensitivity analysis also indicate that the input parameters are not highly correlated among themselves. To this aim, the parameter statistics to look at are the correlation coefficients, $\text{cor}(i,j)$. They are derived from the computed parameter covariances as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Perturbation</th>
<th>$c_{ss}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>1000 kPa</td>
<td>0.01</td>
<td>5.59</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.3</td>
<td>0.01</td>
<td>6.89</td>
</tr>
<tr>
<td>$c$</td>
<td>0</td>
<td>None</td>
<td>-</td>
</tr>
<tr>
<td>$\phi$</td>
<td>40°</td>
<td>0.01</td>
<td>9.58</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.1</td>
<td>0.01</td>
<td>4.28</td>
</tr>
<tr>
<td>$n$</td>
<td>0.4</td>
<td>0.01</td>
<td>5.21</td>
</tr>
<tr>
<td>$G_s$</td>
<td>2.65</td>
<td>0.01</td>
<td>5.80</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.40</td>
<td>0.01</td>
<td>29.43</td>
</tr>
</tbody>
</table>

### Figure 5: Experimental observations and results of the initial MPM simulation at the end of the test.
\[ \text{cor}(i, j) = \frac{\text{cov}(i, j)}{\sqrt{\text{var}(i) \cdot \text{var}(j)}} \]  

(5)

where \( \text{cor}(i, j) \) is the correlation coefficient between parameter \( i \) and parameter \( j \); \( \text{cov}(i, j) \) equal the off-diagonal elements of the variance-covariance matrix; \( \text{var}(i) \) and \( \text{var}(j) \) refer to the diagonal elements of the variance-covariance matrix.

The highest correlation coefficient values are reported for the correlation between parameters \( E \) and \( \phi \) (-0.60), between parameters \( n \) and \( \psi \) (-0.55) and between parameters \( n \) and \( \psi \) (0.47). The absolute values are always significantly lower than 0.8, thus indicating that only mild correlations exist between some of the input parameters.

Based on the results of the sensitivity analysis, it has been chosen to calibrate only the two input parameters to which the model results are most sensitive, i.e. the contact coefficient between the soil and the base of the apparatus and the soil friction angle.

3.3 Calibrated MPM simulation

The values of two model parameters, \( \phi \) and \( \mu_s \), are calibrated adopting the regression analysis algorithm described in section 2.3. To this aim, many different sets of observations have been used, considering that elevation data of the propagating soil are available at four experimental stages corresponding to the following times: 0.32 s, 0.53 s, 0.93 s and 1.5 s (Fig. 2). For each experimental time, the 18 elevation points adopted to describe the soil surface profile along a longitudinal cross-section have been further subdivided in two classes, in relation to whether the observations refer to areas with or without soil. This distinction allows to differentiate between experimental data carrying information of the absolute value of the soil depth at a given location (i.e. observation sets a) and data only reporting the "absence" of soil at that location (i.e. observation sets b).

Six inverse analyses have been performed considering the following observation sets: all the available elevation data (72 observations), type-a elevation data (36 observations), and single stage elevation data from each one of the four experimental stages considered (18 observations). Contrarily to what one may have expected, the best results have not been obtained when all the observations are used. Indeed, the largest objective function reduction refers to the regression analysis conducted using only the soil elevation values at the end of the propagation, i.e. observation sets 4a and 4b (Figure 6).

![Figure 6. Result of model calibration by regression using observations at time t = 1.5 s.](image)

![Figure 7. Comparison between model results and experimental observations.](image)
In particular, the best fit between computed and experimental data is obtained, in this case, by increasing the values of both the parameters being calibrated: from 40° to 43.43 for parameter $\theta$, and from 0.40 to 0.48 for parameter $\mu$. Figure 7 shows the comparison between the observations and the results of the model for all the four experimental stages. The latter are reported for both the initial simulation and the calibrated one, i.e. best performing inverse analysis. The improvement of the model in reproducing the final cross section of the soil mass (i.e. observation time 1.5 s) is manifest.

The calibrated model is almost perfectly reproducing the position of the deposited soil mass and only slightly overpredicting the soil deposition heights. On the contrary, the initial simulation shows a soil mass propagating few centimeters below the tip of the experimental observations as well as an almost total absence of soil along the final part of the slope. Moreover, the calibrated model is also adequately, although not optimally, reproducing the evolution of the soil during propagation (i.e. observation times 0.32 s, 0.33 s, 0.93 s). It means that the information carried by the final position of the soil mass allows the calibration of the important model parameters ($\theta$ and $\mu$) that ensures a good simulation of the time-dependent behaviour of the propagating mass.

4 CONCLUSIONS

The paper presented the calibration by inverse analysis of the MPM model of a debris flow experiment conducted in a small-scale flume. The results of the optimized model demonstrated the ability of a MPM schematization of the test to simulate, in space and time, the behaviour of a dry sand flow. The case study also highlighted the effectiveness of the adopted calibration procedure to detect the input parameter values producing the best fit between experimental observations and simulated soil heights. The role of the observations adopted in the optimization algorithm was also investigated. It showed that the deposited soil heights at the end of the propagation are sufficient to adequately calibrate the numerical model. This should be considered very good news because, differently to what happens for laboratory experiment, it is very difficult to get in-situ observations of debris flows related to the propagating soil mass.

REFERENCES


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